

LESSON 5. 2b

Simplifying Expressions with Rational Exponents and Radicals

Today you will do more of yesterday:

- Use properties of rational exponents to simplify expressions with rational exponents.
- Use properties of radicals to simplify and write radical expressions in simplest form.
- Practice using English to describe math processes and equations

Previous required knowledge:

- When we decide (and why) to use \pm in our answer
- Properties of exponents/radicals
- Powers of 2, 3, 4, 5, 6...

BIG IDEA OF THE DAY!

- Sometimes we need to use absolute value in our answer.

Why?

- Because $\sqrt{\quad}$ (and even roots) are defined to give the **principle root** only as the answer!

What is the trick? How do we know when to use absolute value? When:

1. You are starting with an even root
2. and end with an odd exponent

Example:

$$\begin{aligned}\sqrt{x^3} &= \sqrt[2]{x^3} && \text{Starting with an even root (2)...} \\ &= \sqrt{x^2 \cdot x} && \text{Split up inside} \\ &= \sqrt{x^2} \cdot \sqrt{x} && \text{Break into pieces} \\ &= x \cdot \sqrt{x} && \text{Simplify each piece} \\ &= x^1 \cdot \sqrt{x} && \text{We ended up with an odd power and...} \\ &= |x| \cdot \sqrt{x} && \text{...because we started with an even root need to ensure answer only has the principle root}\end{aligned}$$

***** START WITH AN EVEN ROOT, END WITH AN ODD POWER ... MUST USE ABSOLUTE VALUE *****

Simplify each expression.

a. $\sqrt[3]{64y^6}$

b. $\sqrt[4]{\frac{x^4}{y^8}}$

SOLUTION

Because a variable can be positive, negative, or zero, sometimes absolute value is needed when simplifying a variable expression.

	Rule
When n is odd	$\sqrt[n]{x^n} = x$
When n is even	$\sqrt[n]{x^n} = x $

STUDY TIP

You do not need to take the absolute value of y because y is being squared.



a. $\sqrt[3]{64y^6} = \sqrt[3]{4^3(y^2)^3} = \sqrt[3]{4^3} \cdot \sqrt[3]{(y^2)^3} = 4y^2$

b. $\sqrt[4]{\frac{x^4}{y^8}} = \frac{\sqrt[4]{x^4}}{\sqrt[4]{y^8}} = \frac{\sqrt[4]{x^4}}{\sqrt[4]{(y^2)^4}} = \frac{|x|}{y^2}$

BIG IDEA OF THE DAY!

- Sometimes we need to use absolute value in our answer.

***** START WITH AN EVEN ROOT, END WITH AN ODD POWER ... MUST USE ABSOLUTE VALUE *****

Only time we don't do this is ... if we are told "*assume all variable are positive.*"

Write each expression in simplest form. Assume all variables are positive.

a. $\sqrt[5]{4a^8b^{14}c^5}$

b. $\frac{x}{\sqrt[3]{y^8}}$

c. $\frac{14xy^{1/3}}{2x^{3/4}z^{-6}}$

SOLUTION

$$\begin{aligned} \text{a. } \sqrt[5]{4a^8b^{14}c^5} &= \sqrt[5]{4a^5a^3b^{10}b^4c^5} \\ &= \sqrt[5]{a^5b^{10}c^5} \cdot \sqrt[5]{4a^3b^4} \\ &= ab^2c\sqrt[5]{4a^3b^4} \end{aligned}$$

Factor out perfect fifth powers.

Product Property of Radicals

Simplify.

$$\begin{aligned} \text{b. } \frac{x}{\sqrt[3]{y^8}} &= \frac{x}{\sqrt[3]{y^8}} \cdot \frac{\sqrt[3]{y}}{\sqrt[3]{y}} \\ &= \frac{x\sqrt[3]{y}}{\sqrt[3]{y^9}} \\ &= \frac{x\sqrt[3]{y}}{y^3} \end{aligned}$$

Make denominator a perfect cube.

Product Property of Radicals

Simplify.

$$\text{c. } \frac{14xy^{1/3}}{2x^{3/4}z^{-6}} = 7x^{(1-3/4)}y^{1/3}z^{-(-6)} = 7x^{1/4}y^{1/3}z^6$$

COMMON ERROR

You must multiply both the numerator *and* denominator of the fraction by $\sqrt[3]{y}$ so that the value of the fraction does not change.



Perform each indicated operation. Assume all variables are positive.

a. $5\sqrt{y} + 6\sqrt{y}$

b. $12\sqrt[3]{2z^5} - z\sqrt[3]{54z^2}$

SOLUTION

a. $5\sqrt{y} + 6\sqrt{y} = (5 + 6)\sqrt{y} = 11\sqrt{y}$

b. $12\sqrt[3]{2z^5} - z\sqrt[3]{54z^2} = 12z\sqrt[3]{2z^2} - 3z\sqrt[3]{2z^2} = (12z - 3z)\sqrt[3]{2z^2} = 9z\sqrt[3]{2z^2}$

Homework

Pg 249, #48-81